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## Extended Abstract

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16. ABSTRACT

A Monte Carlo technique for the simulation of two-particle relative diffusion is developed using the Eulerian statistical properties of the turbulence. Numerical results are presented which clarify the role of interparticle velocity correlations and reveal the importance of an Eulerian parameter that is not present in Lagrangian theories of relative diffusion. Data on the absolute diffusion and meandering of a plume in a field of grid-generated wind tunnel turbulence are presented. These data are in good agreement with the Monte Carlo simulations.

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# MONTE CARLO SIMULATION OF TWO-PARTICLE RELATIVE DIFFUSION USING EULERIAN STATISTICS

Extended Abstract

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## 1. INTRODUCTION

Monte Carlo techniques have been used extensively to simulate absolute or total diffusion in a turbulent flow field. However, less effort has been devoted to simulating relative diffusion in spite of its importance in analyzing the statistics of plume meandering, instantaneous plume widths, and concentration fluctuations. Since most models of relative diffusion are based on the statistics of pairs of tracer particles, calculating the trajectories of particle pairs is a key problem in Monte Carlo simulations.

An approximate method for the analysis of relative diffusion has been discussed in several recent publications. In this approach the trajectories of particles released as pairs or in clusters are assumed to be statistically independent, but the initial particle velocities are "conditioned" to account for the effects of interparticle velocity correlations. Due to the independence of the trajectories, this approach is referred to as a one-particle model. It was recently applied to atmospheric diffusion from a continuous point source by Gifford (1982) and to diffusion from a finite-size finite-duration source by Lee and Stone (1983a), hereafter referred to as LSa. It was criticized by Smith (1983), defended by Gifford (1983), and reviewed by Sawford (1984), but its validity has not been resolved in a definitive manner. The model has an obvious inconsistency because it accounts for interparticle velocity correlations at the source but ignores them in the particle trajectory calculations downwind of the source.

In this study we modify the one-particle Monte Carlo simulations of relative diffusion in LSa to account for interparticle velocity correlations both at the source and along the particle trajectories. The interparticle correlations are calculated directly from the Eulerian space-time velocity autocorrelation function. Since the particle trajectories are coupled, this is referred to as a two-particle model. Results from these two-particle simulations are compared with our previous one-particle results to determine the importance of the interparticle correlations downwind of the source. The models are evaluated by comparison with new wind-tunnel data on the absolute diffusion and meandering of a plume in a field of grid-generated turbulence.

## 2. THEORETICAL ANALYSIS

We consider the one-dimensional motion of a pair of tracer particles in a field of stationary homogeneous turbulence as illustrated schematically in Fig. 1. Here t is the time after release of the particles, U is the mean wind velocity, y is the cross-wind coordinate, v is the cross-wind component of the turbulent velocity, and $\ell$ is the particle separation. The velocity of each tracer particle is equal to the local value of the turbulent field velocity v(y,t) along its

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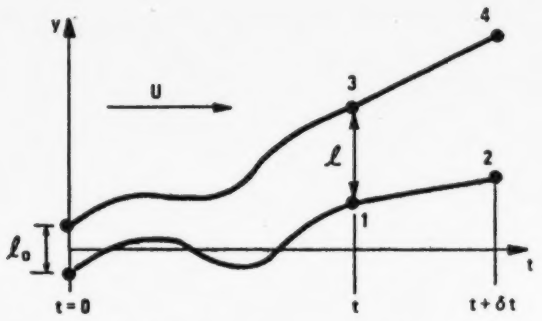


Fig. 1. Schematic of a typical particle-pair trajectory.

trajectory. Therefore, the particle velocities can be simulated using the Eulerian space-time velocity autocorrelation function for the turbulent field, $R_E$. However, it is more convenient to use the Eulerian autocorrelation function $R_C$ in the convective reference frame that moves with the mean wind velocity U. We assume that $R_C$ is an exponential function of separation in space $\zeta$ and time $\tau$; i.e., $R_C(\zeta,\tau) = \exp(-|\zeta|/L)\exp(-\tau/t_C)$, where $t_C$ is the Eulerian integral time scale in the convective reference frame and L is the Eulerian integral length scale. Exponential autocorrelations are physically unrealistic in some ways, but they are consistent with many of the well-known properties of absolute and relative diffusion as discussed by Tennekes (1979) and Sawford (1984). They also permit the time scale $t_C$ and the Lagrangian integral time scale $t_L$ to be simply related to the fixed-frame Eulerian scales $t_E$ and L as derived by Lee and Stone (1983b), hereafter referred to as LSb.

The displacements of the particles from time t to t + δt are assumed to be linear; i.e., $y_2 = y_1 + v_1\delta t$ and $y_4 = y_3 + v_3\delta t$ where $\delta t \ll t_L$. We also assume that the velocities at time t + δt are the sum of correlated and random parts and that the correlated parts can be related to the velocities at time t in a linear manner as follows:

$$v_2 = av_1 + bv_3 + cv_4 + v_2' \qquad (1)$$

and

$$v_4 = dv_3 + ev_1 + fv_2 + v_4' \ , \qquad (2)$$

where $v_2'$ and $v_4'$ are uncorrelated random velocities. The coefficients in these equations and the variances of the random velocities $v_2'$ and $v_4'$ are determined by taking moments and ensemble averages in which the relative positions of points 1, 2, 3, and 4 are held fixed, i.e., Eulerian ensemble averages. The velocity correlations in the resulting equations are set equal to the value dictated by $R_C$. For example, the term $[v_2v_3]/[v^2]$ is set equal to $R_C(\zeta = y_2 - y_3, \tau = t_2 - t_3)$ where [] denote Eulerian ensemble averages. Thus the coefficients have different numerical values at each time step and for each particle-pair trajectory in the Lagrangian ensemble. Details of the evaluation of these coefficients are presented in Lee et al. (1985).

Use of the Eulerian autocorrelation function $R_C$ introduces the parameter $\alpha = \sigma_v t_C/L$ into the problem. This parameter appears in studies of Eulerian-Lagrangian relationships in homogeneous turbulence, e.g., Baldwin and Johnson (1972) and LSb. Based upon their theoretical studies and their analysis of wind-tunnel data, Baldwin and Johnson suggest that α is of order unity and will probably be limited to the range of values $0.3 < \alpha < 5$. In this study we considered the range $0.3 < \alpha < \infty$ since the $\alpha = \infty$ limit corresponds to the frequently used Taylor hypothesis of frozen turbulence in which $L = Ut_E$ (LSb). As a practical matter, the numerical solutions are very near the frozen turbulence limit for $\alpha > 5$.

The relative diffusion $\sigma_R$ and the meandering $\sigma_C$ are defined respectively as the standard deviation of the particle displacements relative to the centroid of the pair of particles and the standard deviation of the centroid displacements relative to the mean wind. The absolute or total dispersion $\sigma_T$ is the standard deviation of the particle displacements relative to the mean wind, and $\sigma_T^2 = \sigma_R^2 + \sigma_C^2$. They are presented in the dimensionless form of $\Sigma^2 = \sigma^2/2[v^2]t_L^2$ as a function of $T = t/t_L$. A comparison of one-particle and two-particle results is presented in Fig. 2 for an initial particle separation of $\ell_o/L = 0.01$ and $\alpha = 1$. Both solutions have an initial region in which $\Sigma_R$ increases as T, followed by an accelerated growth region in which $\Sigma_R$ increases approximately as $T^{3/2}$ in agreement with the inertial range scaling law of Batchelor (1950). All of the curves approach an asymptotic growth rate of $T^{1/2}$ with $\Sigma_R = \Sigma_C = \Sigma_T/(2)^{1/2}$ as T approaches infinity, which is also in agreement with Batchelor's theory. The effect of the interparticle correlation terms in the two-particle model is to delay the onset of the accelerated growth region. The one-particle $\Sigma_R$ values are, at most, a factor of about 3 larger than the two-particle values. The effect of the parameter α on the solutions is presented in Lee et al. (1985).

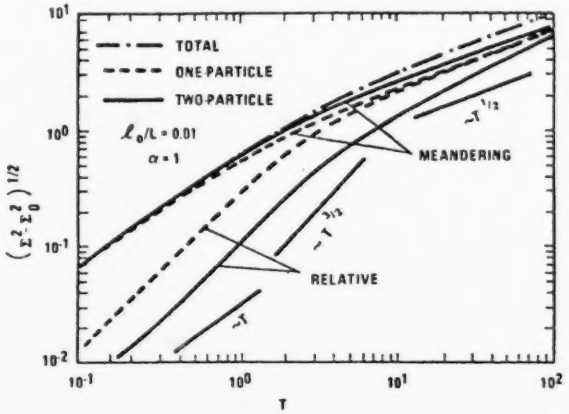


Fig. 2. Comparison of one-particle and two-particle models of meandering and relative diffusion.

## 3. EXPERIMENTAL STUDIES

The theoretical results presented in Section 2 were evaluated using grid-generated turbulence in the meteorological wind tunnel at the EPA Fluid Modeling Facility, which is described by Snyder (1979). A schematic of the experimental arrangement is shown in Fig. 3. The standard grid was modified by the addition of a checkerboard pattern of 20-cm-square Masonite plates with a 40-cm horizontal and vertical spacing between plates in order to produce more meandering of the plume. The mean axial velocity, the turbulence intensities in the axial and vertical directions, and the temporal and spatial correlations of the axial and vertical velocities were measured using hot-wire and hot-film X-array sensors.

The turbulence intensity was nearly uniform in the plane normal to the flow. It decayed slowly in the downwind direction from a value of 0.076 at the source location, $x = 0$, to 0.04 at the downstream end of the experimental region, $x = 6$ m. The mean velocity U was nearly uniform throughout the experimental region with a value of $3.1 \pm 0.1$ m/s. Data on the autocorrelation of the vertical velocity w as a function of the time delay at a fixed point in space were fit with exponential curves to determine the integral time scale $t_E$. Data on the autocorrelation of the axial velocity u and the vertical velocity w as a function of the vertical separation z between two probes were also fit with exponentials to determine the respective integral length scales $L_{uz}$ and $L_{wz}$. These quantities are tabulated in Table 1 for three axial locations downwind of the source. Values of $Ut_E/L_{uz}$ are also tabulated in Table 1. These ratios are very close to unity, which indicates frozen turbulence with $\alpha$ approaching infinity.

The absolute or total diffusion $\sigma_T$ was determined by injecting ethylene ($C_2H_4$) continuously and isokinetically into the flow field through a small-diameter tube as shown schematically in Fig. 3. Plume samples were withdrawn through a sampling rake of four 1.6-mm-diameter tubes, each tube being routed to a separate flame ionization detector. The rake was traversed across the plume vertically at various axial positions downstream of the source. The

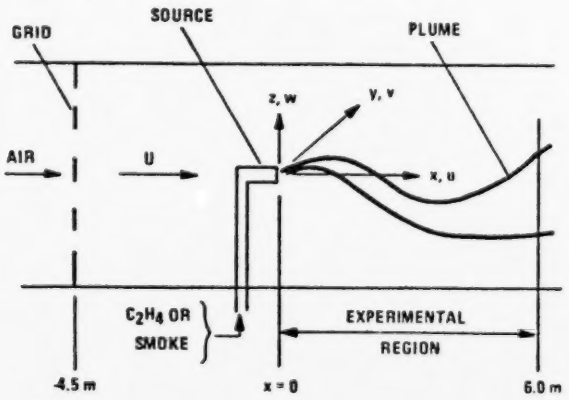


Fig. 3. Schematic of experimental arrangement, side view.

Table 1

Eulerian integral length and time scales along the axis of the wind tunnel ($y = 0$, $z = 0$).

| x (m) | $\sigma_w$ (m/s) | $t_E$ (s) | $L_{wz}$ (m) | $L_{uz}$ (m) | $Ut_E/L_{uz}$ |
|---|---|---|---|---|---|
| 0.5 | 0.207 | 0.0224 | 0.109 | 0.0712 | 0.960 |
| 3.0 | 0.159 | 0.0282 | 0.133 | 0.0858 | 1.002 |
| 6.0 | 0.130 | 0.0321 | 0.144 | 0.0995 | 0.984 |

measured concentration profiles were fit to the Gaussian plume equation to determine the mean and the standard deviation $\sigma_T$ of the distributions. We assumed that $\sigma_y = \sigma_z = \sigma_T$, and this was verified by measuring both vertical and lateral profiles at a few axial locations.

The meandering of the plume centroid $\sigma_C$ was determined by taking instantaneous photographs of a smoke plume, which was injected isokinetically into the tunnel through a small tube as shown schematically in Fig. 3. The plume was photographed at 1-s intervals using a shutter speed of 1/500 s. Negatives of the exposures were digitized using a scanning microdensitometer to obtain vertical profiles of optical density, and the z-coordinate of the plume centroid was obtained by taking the first moment of these profiles. A typical photograph, optical density profiles, and centroid locations are shown in Fig. 4 for $0 < x < 3$ m. Values of $\sigma_C$ for an ensemble of 70 photographs are shown in Fig. 5 along with the $\sigma_T$ values discussed above.

Results of the Monte Carlo simulations for the wind tunnel conditions are also shown in Fig. 5. Values of $\sigma_w$ and $L_{wz}$ measured as a function of distance downwind of the source were used in these simulations, and the turbulence was assumed to be frozen with $\alpha$ and $t_C$ infinitely large. Thus there were no free parameters to adjust in the model. The agreement between the experimental results and the two-particle simulations is very good, but the one-particle model underestimates $\sigma_C$ significantly. We conclude, therefore, that the two-particle model provides a more accurate description of plume meandering and relative diffusion. This experimental evaluation is limited, however, since it consists of only one set of flow conditions that produced frozen turbulence.

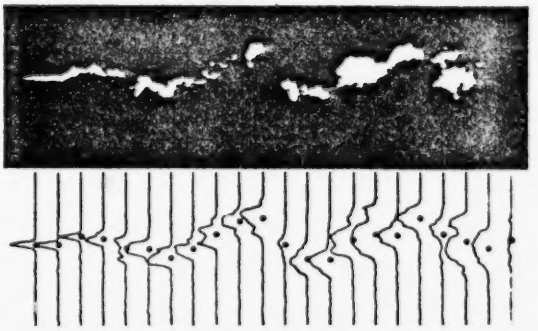

Fig. 4. Instantaneous photograph of the smoke plume with vertical profiles of the optical density and centroid positions over the region $0 < x < 3$ m.

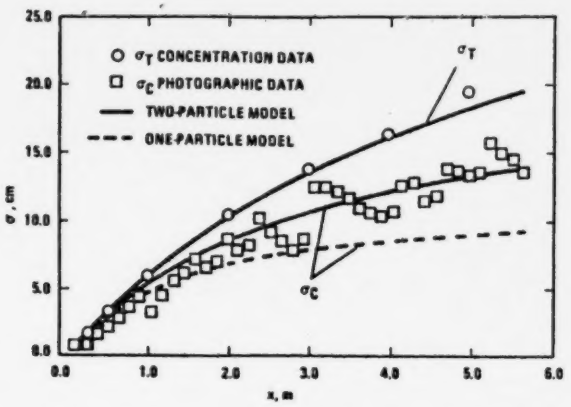


Fig. 5. Comparison of concentration and photographic data with one-particle and two-particle Monte Carlo simulations.

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